OCHA-PP-79
June 1996

Baryogenesis with vector-like quark model in charge transport mechanism

Tomoko Uesugi[1], Akio Sugamoto and Azusa Yamaguchi
Department of Physics, Ochanomizu University, Tokyo 112, Japan

Abstract

The electroweak baryogenesis is studied in the charge transport mechanism with the vector-like quark model. Introducing an extra vector-like up-type quark and a singlet Higgs scalar with the mass of the order of a few hundred GeV, the baryon number generation from the bubble wall is estimated. We show that this scenario is consistent with the measurement of the present baryon to entropy ratio of our universe, if the parameters are in the right region.

1 Introduction

The baryogenesis scenario in the electroweak scale was suggested in order to solve a number of problems in the GUT scale baryogenesis. Within the minimal standard model, however, this electroweak baryogenesis scenario faces other serious problems. First, CP-violation source coming only from the CKM phase is too small to explain the observed baryon to entropy ratio. Second, the electroweak phase transition is of the weak first order one in the minimal standard model, although the strong first order phase transition is

[1]E-mail:tomoko@phys.ocha.ac.jp

required for the baryogenesis to be realized. Third, after the phase transition finishes, the difficult wash out problem exists. In order to solve these serious problems, we have to go beyond the standard model.

Recently, Nelson, Kaplan and Cohen proposed the electroweak baryogenesis scenario called the charge transport mechanism [1]. If the electroweak phase transition is of the first order, bubbles of the broken phase appear in the symmetric phase during the development of the phase transition. The charge transport mechanism is the scenario in which quarks reflect from the expanding bubble wall in a manner that produces net hypercharge flux in the symmetric phase. This hypercharge production is guaranteed by the CP-violating interactions inside the wall. Then the produced hypercharge flux is converted into the baryon asymmetry through the sphaleron transition active in the unbroken phase outside the bubble wall.

In this paper, we study the electroweak baryogenesis with the vector-like quark model. This is a minimal extension of the standard model in which extra SU(2) singlet vector-like quarks and one extra singlet Higgs scalar are added to the standard model. Historically this model was first introduced by Bento, Branco and Parada [2] as one of the simplest spontaneous CP-violation models. The vector-like quark model is very attractive to the electroweak baryogenesis, since it can not only accommodate an extra CP violation source, but also makes the first order phase transition of the electroweak

theory much stronger. The baryogenesis problem with the vector-like quark model was studied by J.McDonald [7] using the spontaneous baryogenesis mechanism which is different from the charge transport mechanism studied in this paper.

2 The vector-like quark model

We consider here an extension of the standard model by adding extra SU(2) singlet quarks U_L and U_R as the fourth generation as well as an extra singlet Higgs scalar S. These extra singlet quarks are called vector-like, since they have left-right symmetric weak isospin and their couplings with the gauge fields become vector-like.

The field content of this model is

$$
\begin{gathered}
(u\ d)^i_L\,, \quad u^\alpha_R, \quad d^i_R, \quad U_L, \quad \phi, \quad S, \\
i = 1,2,3, \quad \alpha = 1,2,3,4
\end{gathered}
$$

where i and α are the generation indices, and ϕ denotes the standard Higgs doublet. We assume that the additional singlet Higgs S is a complex scalar and all the new fields introduced, U_L, $U_R \equiv u^4_R$ and S, are odd under a Z_2 symmetry, whereas the ordinary fields are even.

Under these conditions, the $SU(2)\times U(1)\times Z_2$ invariant Yukawa couplings

are given by

$$L_Y = -\sqrt{2}(\bar{u}\bar{d})^i_L(h_{ij}\tilde{\phi}d^j_R + f_{ij}\phi u^j_R) - \mu\bar{U_L}U_R - \sqrt{2}(f_{i4}S + f^{'}_{i4}S^*)\bar{U_L}u^i_R + h.c.\ , \quad (1)$$

with additional coupling constants μ, f_{i4} and $f^{'}_{i4}$ ($i = 1 \sim 3$).

We consider a scenario in which the extra singlet scalar, S, obtains a vacuum expectation value at the higher energy scale before the electroweak phase transition starts, and later the doublet Higgs, ϕ, obtains the vacuum expectation value during the electroweak phase transition. Therefore after the electroweak phase transition, S and ϕ are able to have the following vacuum expectation values,

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad \langle S\rangle = \frac{V}{\sqrt{2}}e^{i\alpha}. \quad (2)$$

The most general renomalizable $SU(2) \times U(1) \times Z_2$ symmetric scalar potential is as follows,

$$\begin{aligned} V &= V_\phi + V_S + V_{\phi,S}, \\ V_\phi &= \frac{\rho^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2, \\ V_S &= S^*S(a_1 + b_1S^*S) + (S^2 + S^{*2})(a_2 + b_2S^*S) + b_3(S^4 + S^{*4}), \\ V_{\phi,S} &= \phi^\dagger\phi\left[c_1(S^2 + S^{*2}) + c_2S^*S\right], \end{aligned} \quad (3)$$

where all the coefficients are assumed to be real. With the given profile of the expectation values, v and V, expectation value of the phase α can be

obtained by minimizing the potential with respect to α, that is,

$$\alpha = \frac{1}{2}\cos^{-1}\left[-\left(\frac{a_2 + b_2 V^2/2 + c_1 v^2/2}{b_3 V^2}\right)\right]. \tag{4}$$

From this eq.(4) we understand that the value of α can be nonvanishing even for $v = 0$ before the electroweak phase transition starts. Furthermore it can be position-dependent inside the bubble wall, during the electroweak phase transition is undertaking, reflecting the space-dependency of v there. The position-dependent complex phase is the manifestation of the spontaneous CP violation in this model. There is, however, a problem inherent to the spontaneous CP violation, that is, there are 2 types among the 4 independent solutions satisfying eq.(4), having different CP properties. That is, if the phase α_+ is one of the solutions, the phase α_- satisfying $\sin(\alpha_-) = -\sin(\alpha_+)$ is also the solution. Therefore, generation of the bubbles having these different CP phases may cancel the net hyperchage production from the bubble walls. In order to avoid this difficulty, an explicit but tiny CP breaking may be helpful. Following [7], if we replace the factor $(S^2 + S^{*2})$ in the potential $V_{\phi,S}$ to $(e^{i\alpha_0}S^2 + e^{-i\alpha_0}S^{*2})$, then the difference of the potential ΔV between two solutions of opposite signs, α_+ and α_- is roughly $\Delta V \sim O(V^4)\sin(\alpha_0)|\sin(2\alpha_\pm)|$. Let us consider $(-)$ bubbles having less potential energy than the surrounding $(+)$ bubbles by ΔV. In order for the $(-)$ bubbles can grow up to fill the whole universe, the critical radius r_c of the $(-)$ bubble in the surrounding $(+)$ bubbles should be less than the

radius of the universe r_H :

$$r_c \leq r_H \ . \tag{5}$$

The critical radius r_c is the ratio of the surface tension $\sigma \sim O(V^3)$ over the ΔV, $r_c \sim \sigma/\Delta V \sim 1/(V \sin(\alpha_0)|\sin(2\alpha_\pm)|)$. The radius of the universe, r_H, is related to the Hubble constant at temperature T as follows,

$$r_H = H(T)^{-1},$$

where

$$H(T) \sim 20T^2/M_{pl}.$$

Therefore eq.(5) leads to

$$\sin\alpha_0|\sin 2\alpha_\pm| \geq \frac{20T^2}{VM_{pl}} \sim 10^{-16}, \tag{6}$$

where we have chosen $T = 100$ GeV, $V = 500$ GeV and $M_{pl} = 10^{19}$ GeV. Therefore the tiny explicit CP breaking of $O(10^{-16})$ is enough to keep the $(-)$ bubbles having the same CP properties, eliminating the other $(+)$ bubbles.

The shape of the wall $v(z)$ can be obtained from the equation of motion for ϕ. Here, we assume that a wall profile can be determined dominantly from the potential V_ϕ as

$$v(z) = v_0 \left(\frac{1 + \tanh(z/\delta_w)}{2} \right), \tag{7}$$

where we consider only the dependency on the coordinate z, normal to the bubble wall, but we can get the more general solutions by boosting the wall profile (7) in the x-y plane. Here δ_w represents the thickness of the wall.

Now, let us consider the Yukawa couplings of the up-type quarks. As the weak eigen states, we take the bases in which mass matrix of the down-type quarks is diagonalized. In these bases, mass matrix of the up-type quarks can be written as

$$M = \begin{pmatrix} m_{ij} & 0 \\ M_{4j} & \mu \end{pmatrix}, \tag{8}$$

where

$$m_{ij} = f_{ij} v, \quad M_{4j} = (f_{4j} S + f'_{4j} S^*).$$

This mass matrix can be diagonalized by the bi-unitary transformation, that is, by using the two unitary matrices U_L and U_R we have

$$U_L^\dagger M U_R = M_d \equiv \begin{pmatrix} \bar{m}_{ii} & 0 \\ 0 & \bar{M} \end{pmatrix}, \tag{9}$$

where $\bar{m} = (m_u, m_c, m_t)$ and $\bar{M}$ is the vector-like quark mass M_U. We assume that the vector-like quark and the singlet Higgs are heavier than the standard quarks and doublet Higgs.

3 Computation of the hypercharge flux

For simplicity, we consider here only top quark and the vector-like quark in the computation of the hypercharge flux. What we have to do is to solve

the Dirac equation under the position-dependent wall profile and to find the reflection coefficients of the quarks from the wall. The two-generations' up-quark mass matrix is

$$M = \begin{pmatrix} fv(z) & 0 \\ FVe^{i\alpha(z)} + F^{'}Ve^{-i\alpha(z)} & \mu \end{pmatrix}, \tag{10}$$

where $f = f_{33}$, $F = f_{43}$ and $F^{'} = f^{'}_{43}$. Using this z-dependent mass matrix, we will write the Dirac equation in the chiral bases. For the stationary state ($\psi(t,z) = \psi(z)e^{iEt}$), the Dirac equation reads

$$\begin{aligned} i\frac{\partial}{\partial z}\begin{pmatrix} \psi_{1R}(z) \\ \psi_{3L}(z) \end{pmatrix} &= -\begin{pmatrix} E & M^{\dagger} \\ M & -E \end{pmatrix}\begin{pmatrix} \psi_{1R}(z) \\ \psi_{3L}(z) \end{pmatrix} \\ i\frac{\partial}{\partial z}\begin{pmatrix} \psi_{4L}(z) \\ \psi_{2R}(z) \end{pmatrix} &= -\begin{pmatrix} E & M \\ M^{\dagger} & -E \end{pmatrix}\begin{pmatrix} \psi_{4L}(z) \\ \psi_{2R}(z) \end{pmatrix} \end{aligned} \tag{11}$$

where $\psi_{L(R)}(z)$ has two components, corresponding to the top and the vector-like quarks.

We have solved this Dirac equation numerically as a system of ordinary differential equations with respect to z, without any approximations. The reflection coefficient from the right-handed j-quark to the left-handed i-quark, $R^{ij}(E)$, and from the left-handed j-quark to the right-handed i-quark, $\bar{R}^{ij}(E)$, are given by

$$R^{ij}(E) = \frac{\psi^{i}_{3L}(z_0)}{\psi^{j}_{1R}(z_0)} \tag{12}$$

$$\bar{R}^{ij}(E) = \frac{\psi^{i}_{2R}(z_0)}{\psi^{j}_{4L}(z_0)} \tag{13}$$

where i and j represent the generations, E is the incident energy and z_0 denotes the position of the boundary between the wall and the symmetric phase. Using these reflection coefficients, the hypercharge flux can be written as

$$\begin{aligned} f_Y &= \int_0^\infty dk_L \int_0^\infty k_T dk_T \left(\frac{1}{4\pi^2\gamma}\right) \\ & \sum_{i,j}\left[(\Delta Y)_{j\to i}|R_{ij}(k_L)|^2 + (\Delta Y)_{j\to i}|\bar{R}_{ij}(k_L)|^2\right] \\ & \times\left(f^s(E^s)_j - f^b(E^b)_i\right), \end{aligned} \tag{14}$$

with

$$\begin{aligned} f^s(E)_i &= \frac{k_L}{\sqrt{k_L^2+k_T^2+M_i^{s2}}\,(1+\exp(E^s/T))}, \\ f^b(E)_i &= \frac{k_L}{\sqrt{k_L^2+k_T^2+M_i^{b2}}\,(1+\exp(E^b/T))}, \\ & E^s = \gamma\left(\sqrt{k_L^2+k_T^2+M_i^{s2}} - v_w k_L\right), \\ & E^b = \gamma\left(\sqrt{k_L^2+k_T^2+M_i^{b2}} + v_w k_L\right), \end{aligned} \tag{15}$$

where $(\Delta Y)_{j\to i}$ represents the change of the hypercharge from j-th quark to the i-th quark, γ is the γ-factor $1/\sqrt{1-v_w^2}$ corresponding to the wall velocity v_w, k_T and k_L are transverse and longitudinal components of the momentum in the wall rest frame, and E^s (E^b) and M^s (M^b) are, respectively, the energy and the diagonalized mass of quark in the thermal frame of the symmetric (broken) phase. The integral over k_T can be done analytically. Then we have

$$f_Y = \int_0^\infty dk_L \left(\frac{k_L}{4\pi^2\gamma^2}\right)\sum_{i,j}\left[(\Delta Y)_{j\to i}|R_{ij}(k_L)|^2 + (\Delta Y)_{j\to i}|\bar{R}_{ij}(k_L)|^2\right]$$

$$\times\left[\log\left(1+\exp\left[-\gamma(k_L - v_w\sqrt{k_L^2 - M_j^{s2}})/T\right]\right)\right.$$
$$\left. - \log\left(1+\exp\left[-\gamma(k_L + v_w\sqrt{k_L^2 - M_i^{b2}})/T\right]\right)\right]. \tag{16}$$

We have to give the phase transition temperature and the masses of the quarks in the broken phase for the calculation of the hypercharge flux. We set the phase transition temperature $T = 100$ GeV, and the mass of the top quark $m_t = 174$ GeV. The wall width is considered roughly the order of the phase transition temperature, $\delta_w^{-1} \sim T \sim 100$ GeV, but we have estimated here the hypercharge flux for the various choices of the wall width. The experimental constraints for the mass of the vector-like quark is not so strict [6]. We have tried the cases of $M_U = 500$ GeV and 300 GeV. Most of the parameters in the scalar potential V in eq.(3) are outside of the experimental verification. Therefore we can choose freely a and b with a restriction of $a+b=1$ and determine the position-dependent α as $\cos 2\alpha = a + b(v(z)/v_0)^2$. The restriction above means the disappearance of the spontaneous CP violation phase after the electroweak phase transition ends. We also introduce m_{com} and h defined by $m_{com} = (F + F^{'})V$ and $h = F^{'}/F$. Then, in the case of $M_U = 500$ GeV, $m_{com} = 300$ GeV, and $v_w = 0.5$, we have $f_Y = O(10^{-6\sim-7})$ for the various choices of $(b, h) = (1.0 \sim 2.0, 0.01 \sim 0.5)$. The value $b = 2.0$ corresponds to the case that the complex phase moves from zero to $\pi/2$ inside the wall. If the value h becomes smaller, the imaginary part becomes larger than the real part in the off-diagonal term. The value $h = 1.0$ corresponds

to the real off-diagonal term.

If we change the off-diagonal value, m_{com}, of the mass matrix, we have obtained the following results for $M_U = 300$ GeV, $v_w = 0.1$, $h = 0.01$, and $b = 2.0$: $f_Y = O(10^{-8\sim-6})$ corresponding to the values of $m_{com} = 0.0 \sim 1.2$. (see Fig.1)

Fig.2 shows the wall width dependence of the hypercharge flux in the case of $M_U = 300GeV$ and $M_U = 500GeV$. We can see that the hypercharge flux takes the larger value for the thinner wall.

4 Generation of the baryon asymmetry from the hypercharge flux

Following the method of Nelson, Kaplan and Cohen [1], we compute the partial derivative of the free energy with respect to the baryon number. If the time scale of the baryon number violating sphaleron transition is much longer than that of the weak interactions between reflected quarks, 10^{-26} $[s]$, we will be able to think the process of the sphaleron transition is not in the thermal equilibrium within the time scale of the weak interactions. Then, the baryon number and the lepton number will conserve during the time scale. In that situation we can introduce the chemical potentials corresponding to hypercharge $Y/2$, charge Q, baryon number B, $B-L$ (L : lepton number), and $B^{'} = (B_4 + B_3)/2 - (B_2 + B_1)/2$, where B_i is the baryon number of

the i-th generation. We have neglected the mixing between the lighter (1st and 2nd) generations and the heavier (3rd and 4th) generations, but this is a quite reasonable assumption. Using these chemical potentials, we can obtain the relation between the chemical potential of the baryon number μ_B and the hypercharge density $\rho_{Y/2}$:

$$\begin{aligned} \rho_{Y/2} &= \frac{\mu_B}{24\beta+1}\left[-2(24\beta+\frac{8}{3})(8\beta+1)+(24\beta+1)(16\beta+1)\right]\frac{T^2}{3} \\ &\sim -\frac{13}{9}\mu_B T^2 \ , \end{aligned} \qquad (17)$$

where

$$\beta = \frac{1}{4\pi^2}\int_0^\infty dx \frac{x^2}{1+\cosh\sqrt{x^2+M_U^2}} \ . \qquad (18)$$

One can easily see that the β is small enough to be neglected as long as we set the mass of the vector-like quark M_U at a few hundred GeV . Then the net baryon number production is

$$\begin{aligned} \rho_B &= \frac{\Gamma_{sph}}{T}\int dt \frac{\partial F}{\partial B} \\ &= \frac{9\Gamma_{sph}}{13T^3}\int_{-\infty}^{z/v_w} dt \rho_{Y/2}(z-v_w t) \ , \end{aligned} \qquad (19)$$

where $\rho_{Y/2}(x)$ denotes the hypercharge density at the position which is located at the distance x from the bubble wall. $\Gamma_{sph} = k(\alpha_W T)^4$ is the sphaleron transition rate of the symmetric phase, where $\alpha_W = g^2/4\pi$, and k is a numerical factor which has been estimated by J.Ambjørn et al. about

$0.1 \sim 1.0$ [5]. The integral over t can be estimated approximately by the transport time of the reflected quarks τ_T as follows:

$$\begin{aligned} \int_{-\infty}^{z/v_w} dt \rho_{Y/2}(z - v_w t) &= \frac{1}{v_w} \int_0^{\infty} dx \rho_{Y/2}(x) \\ &\sim \frac{f_Y \tau_T}{v_w} , \end{aligned} \tag{20}$$

and the baryon to entropy ratio becomes

$$\begin{aligned} \frac{\rho_B}{s} &= \frac{9}{13} \frac{k \alpha_W^4 T}{s} \frac{f_Y \tau_T}{v_w} \\ &\sim 10^{-8} k \frac{(\tau_T T)}{v_w} (f_Y / T^3). \end{aligned} \tag{21}$$

Then to explain the present baryon to entropy ratio, we need $f_Y \sim 10^{-5 \sim -6} T^3$, since the typical transport time of the weak scale is roughly $O(10^{1 \sim 3} T^{-1})$. The solutions of $f_Y \sim 10^{-5 \sim -6} T^3$ have been found in the charge transport scenario with the vector-like quark model in the last section, so that the scenario can be a good candidate of the model of the baryogenesis.

5 Conclusion

In this paper, the charge transport scenario of the baryogenesis has been studied using the vector-like quark model. We have adopted the model in which extra left-right symmetric, $SU(2)$ singlet up-type quarks U_L and U_R, and a $SU(2)$ singlet Higgs scalar S are added to the standard model. The possible Higgs potential for the ordinary doublet Higgs ϕ and the singlet S

admits the spontaneous CP violation in which the phase α of S takes the non-vanishing vacuum expectation value. During the first order phase transition bubbles of the broken phase are generated, and the phase α becomes position dependent inside the bubble walls. A tiny but explicit CP violation should be, however, incorporated in order to avoid the difficulty of having two types bubbles. In the presence of this explicit CP breaking, one type of the bubbles creating the baryon number through the charge transport mechanism, remains, whereas the other type of the bubbles decreasing it, disappears.

By solving the Dirac equation numerically without any approximations, we have estimated the rate of the hyperchage production from the bubble walls, in which the presence of the position-dependent CP phase in the mass mixing between the top quark and the extra up-type one is important. The evaluated hypercharge flux from the wall f_Y depends on the various unknown parameters. However, with the top quark mass of 174 GeV, the vector-like quark mass of 500 GeV or 300 GeV, and the phase transition temperature of 100 GeV, we have solutions giving $f_Y \sim 10^{-6}$ for the value of the wall width $\delta_W = (100GeV)^{-1}$. It is the reasonable value for reproducing the baryon to entropy ratio of $10^{-10\sim-11}$. For the transition of the hypercharge to the baryon number, the usual discussion has been extended so as to incorporate the extra vector-like up-type quark and the singlet Higgs, in which the

thermal equilibrium by the weak interactions is violated by the sphaleron transition.

The phase transition dynamics when singlet Higgs is introduced has not been given here. Some of the results can be found by using the 1-loop approximation and the high temperature expansion [4] on how the first order phase transition becomes stronger in our case [9], but the more detailed study should be required. In the real development of the first order phase transition, fusion effect of the bubbles including the temporal change of the expanding velocity of the bubbles, should be taken into account [8]. This kind of analysis is difficult, but very important. Also the detailed analysis should be given on the disappearance of the unwanted bubbles having the different CP properties. In the analysis of estimating the final value for the baryon to entropy ratio, we need to know the exact value of the sphaleron transition rate, or the coefficient k in eq.(21), as well as the transport time τ_T for the various quarks. Improvement on the estimation of these parameters are also necessary.

Acknowledgment

We would like to thank J. Arafune, K. Yamamoto and K. Funakubo for their advice. We are also grateful to I. Watanabe and H. Aoki for their useful discussions.

This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (No.08640357).

References

[1] A. E. Nelson, D. B. Kaplan, and A. G .Cohen, Nucl. Phys. **B373**(1992),453; A. G. Cohen, D. B. Kaplan and A. E. Nelson, Ann. Rev. and Part. Sci., **43**(1993),27

[2] L. Bento, G. Branco, and P. A. Parada, Phys. Lett. **B267**(1991),95

[3] L. Bento, G. Branco, Phys. Lett. **B245**(1990),599

[4] G. W. Anderson and L. J. Hall, Phys. Rev. **D45**(1992),2685;

[5] J. Ambjørn, T. Askgaard, H. Porter, and M. E. Shaposhnikov, Nuc. Phys. **B353**(1991),346;

[6] V. Barger, M. S. Barger, and R. J. N. Phillips, Phys. Rev. **D52**(1995),1663

[7] J. McDonald, Phys. Lett. **B357**(1995),19;
J. McDonald, Phys. Rev. **D53**(1996),645

[8] A. Yamaguchi, and A. Sugamoto, Mod. Phys. Lett. **A9**(1994),2599

[9] A. Yamaguchi, A. Sugamoto, and T. Uesugi (in preparation)

Figure Captions

1. Fig.1: The wall width dependence of the hypercharge flux in the case of $M_U = 500$ GeV and $m_{com} = 300$ GeV for $b = 2.0$, $h = 0.01$, $v_w = 0.1$ and $\delta_w = T^{-1}$ (solid line). The case of $M_U = 300$ GeV and $m_{com} = 100$ GeV is also shown (dashed line).

2. Fig.2: The hypercharge flux as a function of $m_{com} = 0.0 \sim 1.2$ GeV for $M_U = 300$ GeV. Other parameters are the same as in Fig.1.

Fig.1

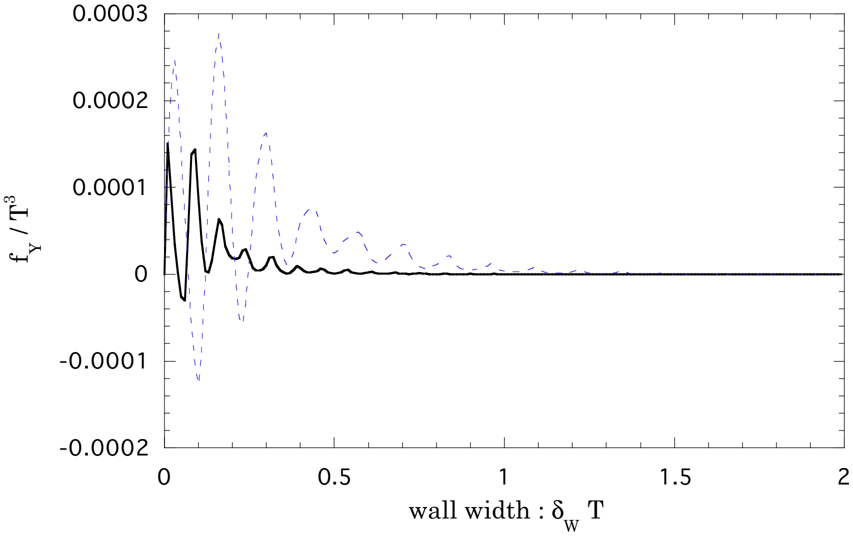
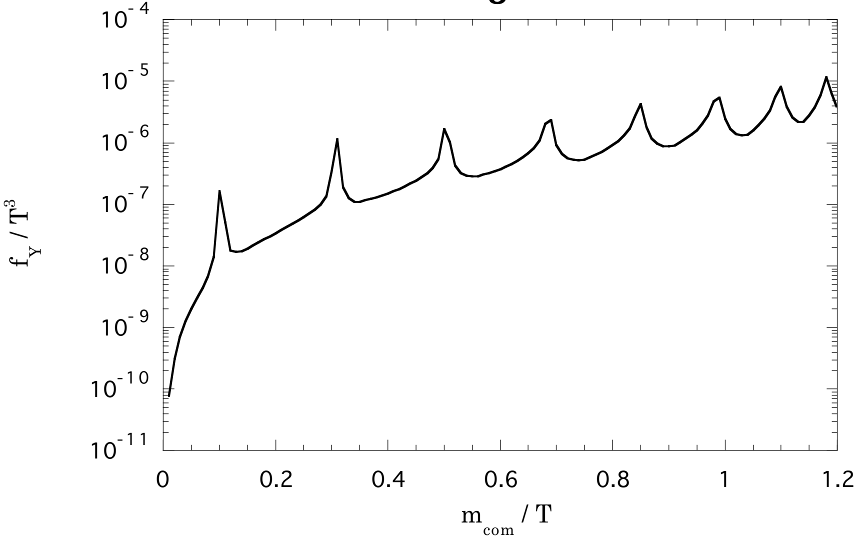

0.0003
0.0002
0.0001
0
-0.0001
-0.0002
f_Y / T^3
0
0.5
1
1.5
2
wall width : $\delta_W T$

Fig.2


10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
f_Y / T^3
0
0.2
0.4
0.6
0.8
1
1.2
m_{com} / T